Problem 1

1) G(x) = ∑∞ k=0 (3x)^k – 1

G(x) = 1/(1-3x) – 1

2) G(x) = ∑∞ k=0 n·x^2k

(1-x^2)G(x) = ∑∞ k=0 x^2k = 1/(1-x^2)

G(x) = 1/(1-x^2)^2

3) G(x) = ∑∞ k=0 C(n+3, 3)·x^k

(1-x)G(x) = ∑∞ k=0 C(n+2, 2)·x^k

(1-x)^2·G(x) = ∑∞ k=0 (k+1)·x^k = 1/(1-x)^2

G(x) = 1/(1-x)^4

Problem 2

1) 1+x / (1-x)^2 = (1+x) ∑∞ k=0 (k+1)·x^k

an = n+1 + n = 2n+1

2) x^2+x / (1-x)^3 = (x^2+x) ∑∞ k=0 C(k+2, k)·x^k

an = C(n, n) + C(n+1, n) = 1 + n-1 = n

an = C(n, n-2) + C(n+1, n-1) = n(n-1)/2 + (n+1)n/2 = n^2

3) 1+x / 1-x^2 = 1 / 1-x = ∑∞ k=0 C(n, n) k·x^k

an = 1^n = n

4) 1-x-x^2 = (1-α1x)(1-α2x), 其中α1 = 1+√5 / 2, α2 = 1-√5 / 2.

G(x) = x / 1-x-x^2 = x / (1-α1x)(1-α2x) = c1 / 1-α1x + c2/1-α2x

x = c1(1-α2x) + c2(1-α1x), c1 = 1/√5, c2 = -1/√5

G(x) = 1/√5 (1 / 1-α1x – 1 / 1-α2x) = 1/√5 ∑∞ k=0 (α1^k·x^k – α2^k·x^k)

an = 1/√5 (α1^n – α2^n) = 1/√5 ( (1+√5 / 2)^n – (1-√5 / 2)^n )

Problem 3

给定系数ak对应的生成函数为f(x) = a0 + a1x + a2x^2 + …,

函数1 / 1-x = ∑∞ k=0 x^k = 1 + x + x^2 + …, 对应系数ck = 1,

则函数1 / 1-x f(x) = ∑∞ k=0 (∑k i=0 ai·ck-i) x^k, 对应系数bk = ∑k i=0 ai

Problem 4

令G(x) = ∑∞ k=0 (k+1)^2·x^k,

f(x) = (1-x)G(x) = ∑∞ k=0 (2k+1)·x^k

由Problem 2 (1)可得对函数1+x / (1-x)^2, x^n 的系数为n+1 + n = 2n+1

f(x) = 1+x / (1-x)^2, 则G(x) = 1+x / (1-x)^3 – k^2·x^(k+1) / (1-x).

1 / 1-x G(x)对应的系数bn-1 = ∑ n-1 i=0 ai = 1^2+2^2+3^2+…+n^2.

h(x) = 1+x / (1-x)^4 = (1+x) ∑∞ k=0 C(4+k-1, k)·x^k.

x^n-1项的系数为C(n+2, n-1) + C(n+1, n-2) = C(n+2, 3) + C(n+1, 3)

= (n+2)(n+1)n / 6 + (n+1)n(n-1) / 6 = n(n+1)(2n+1) / 6.

Problem 5

~~1, 2, 3, 1+2 = 3, 1+3 = 4, 2+3=5, 1+2+3 = 6, 可以贴出6种不同数值的邮资.~~

G(x) = (x^0 + x^1 + x^2 …) (x^0 + x^2 + x^4 …) (x^0 + x^3 + x^6 …)

G(x) = 1 / 1-x · 1 / 1-x^2 · 1 / 1-x^3 = 1 / (1-x)(1-x^2)(1-x^3).

Problem 6

四位数能被2整除则个位只能是2, 等同于3个1, 1个2, 5个3这九个数字

能够构成多少个三位数. 111; 333; 112 121 211; 332 323 233;

113 131 311; 331 313 133; 123 132 213 231 312 321, 共20个.

Problem 7

设G(x)是序列{ak}的生成函数, G(x) = ∑∞ k=0 ak·x^k

(1-4x+4x^2)G(x) = a0 + a1x – 4a0x = 1, 则G(x) = 1 / (1-2x)^2.

G(x) = ∑∞ k=0 C(2+k-1, k)·2^k·x^k, ak = C(k+1, 1)·2^k = (k+1)·2^k.

Problem 8

2+13 = 3+12 = 4+11 = 5+10 = 6+9 = 7+8 = 15, 共6组.

从每个组合随机抽取一个数得6个, 7>6, 则至少有一组两个数都被抽中.

Problem 9

从8门课程选择5门, 有C(8, 5) = C(8, 3) = 8×7×6÷3÷2÷1 = 56种选法.

至少有10名学生的学习计划相同, 则最少有56×9+1 = 505名学生.

Problem 10

任何的有理数可以表示为x=a/b, 其中a∈Z, b∈Z, a, b互质且b≠0.

任意整数被b除的余数一定在{0, 1, …, b-1}中, 共有b种可能.

对 a进行c = a // b, a = (a % b)\*10这一操作, 重复b+1次,

根据鸽笼原理可得, b+1次操作中, a的取值至少会有一次重复.

若这些取值中含有0, x是有限小数, 否则x从取值重复的一位开始循环.